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A GENERAL COVARIANT JUMP CONDITION ACROSS A BUBBLE WALL AND THE QCD PHASE TRANSITION *

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Abstract

Covariant jump conditions across a bubble wall, including surface tension and dissipation, for a nongravitational moving surface have been derived. Special situations, such as those associated with a domain wall, a dust shell, and a spherical boundary, are discussed. In particular, the cosmological QCD phase transition has been examined as a specific application of the general formalism. The possible mechanism of energy transport during the phase transition is studied, and a critical condition for neutrino conduction to dominate over the hydrodynamic flow has been obtained.

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1. Introduction

Bubble dynamics has become one of the more active topics at the boundary of particle physics and cosmology. For example, just as in condensed-matter systems, many physical processes in cosmology may be associated with a possible first order phase transition, including the inflationary stage of the early universe[1] and the possibly related GUT(grand unified theory) phase transition.[2] Furthermore, some have even speculated that the electro-weak phase transition,[3] the QCD phase transition,[4] and even perhaps the formation mechanism of galaxies[5] in the later stages of the expansion of the universe may be related to a first order transition. First order phase transitions are appropriately described by bubble dynamics, because any first order phase transition has to be accompanied by an evolution of the new phase bubbles from their formation, to propagation, to their coalescence. Thus a thorough investigation of bubble dynamics is required for an understanding of the dynamics of the phase transition since, in that transition process, the dynamics of the boundary, the nucleation rates of the bubble, and the mechanism of energy transport through the boundary will be affected directly by dissipative processes and the surface tension, if the bubble is small.

Several authors[6] have recently studied bubble dynamics in the context of general relativity. However, some problems remain: 1. The Gauss-Codazzi formalism[7] is more sophisticated than is necessary and thus can be awkward for the treatment of the problem without gravity, as in the QCD case of a heavy ion collision. 2. The ability of dissipative processes to carry latent heat away from the bubble's surface is usually completely ignored. For the study of the nucleation process of a new phase bubble, this may lead to problems because dissipation may become important in the bubble propagation and affect the shape of the phase boundary. 3. The effects of the surface tension on the motion of the phase boundary are usually not considered.

In this research, we are attempting to derive a covariant jump condition across the bubble wall, including surface tension and possible dissipation, for a surface moving in any manner, but without gravity. We then obtain a general equation of motion for a boundary moving in a background (not vacuum), in the presence of dissipative processes. Further, we will take the QCD phase transition as one of the applications and investigate the mechanism of energy transport through the boundary during an assumed first order QCD phase transition. Finally, we identify a critical condition that determines which energy transport mechanism–hydrodynamic flow or neutrino conduction– is dominant in the expanding bubbles or shells. We are well aware that recent lattice gauge calculations and other considerations[8] favor a 2nd order QCD transition (or even no phase transition at all) and thus make a first order QCD transition unlikely. However, because the possible consequences of a potential first order QCD transition are so dramatic, (particularly for cosmology), we nevertheless feel the study is worthwhile and may teach us about first order transitions in general, even if the QCD transition itself turns out to be at most weakly first order.

2. The Formalism

Let us assume that a plasma is undergoing a first order phase transition and that the two phases of the plasma are separated by a thin wall. The equation of motion of the plasma is described by the fluid equation

$$\nabla_\mu T^{\mu\nu} = 0, \tag{2.1}$$

where $T^{\mu\nu}$ is the energy-momentum tensor of the system.

For the system we are considering, the energy-momentum tensor consists of three parts: one component, denoted by $T^{\mu\nu}_{(0)}$, describes the perfect fluid; the other two terms, $\triangle T^{\mu\nu}$ [9] and $T^{\mu\nu}_{\rm wall}$, correspond, respectively, to the contributions arising from the dissipative processes and the thin wall. A general form of $T^{\mu\nu}$ can be written as

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + \triangle T^{\mu\nu} + T^{\mu\nu}_{\rm wall}, \tag{2.2}$$

where

$$T^{\mu\nu}_{(0)} = (e+p)\,u^\mu u^\nu + pg^{\mu\nu}, \tag{2.3}$$

$$\triangle T^{\mu\nu} = -\eta H^{\mu\alpha}H^{\nu\beta}\omega_{\alpha\beta} - \xi H^{\mu\nu}\partial_\alpha u^\alpha - \chi(H^{\mu\alpha}u^\nu + H^{\nu\alpha}u^\mu)Q_\alpha, \tag{2.4}$$

$$\omega_{\mu\nu} \equiv \partial_\mu u_\nu + \partial_\nu u_\mu - \frac{2}{3}g_{\mu\nu}\,\partial_\alpha u^\alpha,$$

$$H^{\mu\nu} \equiv g^{\mu\nu} + u^\mu u^\nu, \qquad Q_\mu \equiv \partial_\mu + Tu^\alpha\partial_\alpha u_\mu,$$

and

$$T^{\mu\nu}_{\rm wall} = \sigma\Gamma\delta(\mathbf{x} - \mathbf{X})S^{\mu\nu}, \tag{2.5}$$

$$S^{\mu\nu} \equiv \sigma U^\mu U^\nu + \tau(h^{\mu\nu} + U^\mu U^\nu). \tag{2.6}$$

For a domain wall $\tau \equiv -\sigma$ ($\sigma =$ constant), and for a dust shell, $\tau \equiv 0$ ($\sigma\Sigma^2 =$ constant).

In the above expressions and throughout this discussion, we adopt the convention in which the speed of light $c = 1$ and the space-time metric $g^{\mu\nu} = (-1, 1, 1, 1)$. We use e as the energy density, p as the pressure, and $u^\mu \equiv \gamma(1, \mathbf{v})$ as the four-velocity of the fluid, where

$\mathbf{v}$ is the local fluid velocity, and as usual $\gamma \equiv (1-\mathbf{v}^2)^{-1/2}$. Correspondingly, $U^\mu \equiv \Gamma(1, \mathbf{V})$ represents the four-velocity of any observer whose world line lies within the thin wall, and $\mathbf{V}$ is the velocity of the wall, $\Gamma \equiv (1-\mathbf{V}^2)^{-1/2}$. The coefficients η, ξ, χ are the shear viscosity, the bulk viscosity, and the heat conductivity, respectively, the expressions for which are given below[9]:

$$\eta = \frac{4}{15} bT^4 \tau, \tag{2.7}$$

$$\xi = 4bT^4\tau \left[\frac{1}{3} - (\frac{\partial p}{\partial e})_n\right]^2, \tag{2.8}$$

$$\chi = \frac{4}{3} bT^3 \tau. \tag{2.9}$$

Here, b is a constant and given by

$$\begin{aligned} b =& a \quad \text{(photons)} \\ =& \frac{7}{8} a \quad \text{(neutrinos).} \end{aligned} \tag{2.10}$$

where a is the usual Stefan-Boltzmann constant and τ is the mean free time of the radiated quanta. $\omega_{\mu\nu}$ is the shear tensor, Q_μ is the heat flow vector, $H^{\mu\nu}$ is the projection tensor, and σ is the surface energy density of the thin wall. The term $h^{\mu\nu} \equiv g^{\mu\nu} - n^\mu n^\nu = (-1, 1, 1, 0)$ is the three-metric in the local frame of any observer moving within the surface, where n^μ is a unit vector normal to the hypersurface. $\mathbf{X}$ and Σ, respectively, designate the position and the surface area of the thin wall.

3. The Jump Conditions

We now expand Eq.(2.1) into two components: $\nu = 0$ (time-component) and $\nu = i, j$ (space-component, i, j = 1, 2, 3). The former ($\nu = 0$) yields the equation for energy conservation and the later ($\nu = j$) presents the equation for momentum conservation. If the system is in a steady state or equilibrium, the time derivatives in the two components vanish. The jump conditions in a general frame are then obtained by integrating across the thin wall, i.e.,

$\nu = 0$:

$$\begin{aligned} &\frac{\partial X^j}{\partial x^i}\left[w u^i u^0 - \chi \frac{\partial T}{\partial x^i} - \xi \frac{\partial u^i}{\partial t}\right]_{X_-}^{X+} \\ &+ \frac{\partial}{\partial X^i}\Big[\Gamma\big(\sigma U^i U^0 + \tau(h^{i0} + U^i U^0)\big)\Big]_{\mathbf{x}=\mathbf{X}} = 0, \end{aligned} \tag{3.1}$$

and

$\nu = i, j$:

$$\frac{\partial X^j}{\partial x^i}\Big[wu^iu^j + pg^{ij} - \eta(\frac{\partial u_i}{\partial x^j} + \frac{\partial u_j}{\partial x^i} - \frac{2}{3}\nabla\cdot\mathbf{V}\delta_{ij}) - \xi\nabla\cdot\mathbf{V}\delta_{ij}\Big]_{X_-}^{X^+} + \frac{\partial}{\partial X^i}\Big[\Gamma\big(\sigma U^iU^j + \tau(h^{ij} + U^iU^j)\big)\Big]_{\mathbf{x}=\mathbf{X}} = 0. \tag{3.2}$$

where $w \equiv e + p$ is the enthalpy density of the fluid and T is the temperature.

Let us now look at specific cases,

i) Domain wall: $\tau = -\sigma$. For this condition, the above jump conditions become

$$\frac{\partial X^j}{\partial x^i}\Big[wu^iu^0 - \chi\frac{\partial T}{\partial x^i} - \xi\frac{\partial u^i}{\partial t}\Big]_{X_-}^{X^+} + \frac{\partial}{\partial X^i}\Big[-\Gamma\sigma h^{i0}\Big]_{\mathbf{x}=\mathbf{X}} = 0 \tag{3.3}$$

and

$$\frac{\partial X^j}{\partial x^i}\Big[wu^iu^j + pg^{ij} - \eta(\frac{\partial u_i}{\partial x^j} + \frac{\partial u_j}{\partial x^i} - \frac{2}{3}\nabla\cdot\mathbf{V}\delta_{ij}) - \xi\nabla\cdot\mathbf{V}\delta_{ij}\Big]_{X_-}^{X^+} + \frac{\partial}{\partial X^i}\Big[-\Gamma\sigma h^{ij}\Big]_{\mathbf{x}=\mathbf{X}} = 0. \tag{3.4}$$

ii) Dust shell: $\tau \equiv 0$. Here, the jump conditions take the form

$$\frac{\partial X^j}{\partial x^i}\Big[wu^iu^0 - \chi\frac{\partial T}{\partial x^i} - \xi\frac{\partial u^i}{\partial t}\Big]_{X_-}^{X^+} + \frac{\partial}{\partial X^i}\Big[-\Gamma\sigma U^iU^0\Big]_{\mathbf{x}=\mathbf{X}} = 0 \tag{3.5}$$

and

$$\frac{\partial X^j}{\partial x^i}\Big[wu^iu^j + pg^{ij} - \eta(\frac{\partial u_i}{\partial x^j} + \frac{\partial u_j}{\partial x^i} - \frac{2}{3}\nabla\cdot\mathbf{V}\delta_{ij}) - \xi\nabla\cdot\mathbf{V}\delta_{ij}\Big]_{X_-}^{X^+} + \frac{\partial}{\partial X^i}\Big[\Gamma\sigma U^iU^j\Big]_{\mathbf{x}=\mathbf{X}} = 0. \tag{3.6}$$

iii) Spherical boundary:

For a spherical domain wall ($\tau = -\sigma$), the jump conditions can be written

$$\Big[w\gamma^2 v\Big]_{R_-}^{R^+} = \chi\frac{\partial T}{\partial r} + \xi\frac{\partial(\gamma v)}{\partial t} \tag{3.7}$$

and

$$\Big[w\gamma^2v^2 + p\Big]_{R_-}^{R^+} + \frac{1}{R^2\dot{R}}\frac{d}{dt}(\Gamma\sigma R^2) = (\frac{1}{3}\eta + \xi)\frac{\partial(\gamma v)}{\partial r}. \tag{3.8}$$

Here, we have taken $\mathbf{v} = v\mathbf{e_r}$, $\mathbf{V} = \dot{R}\mathbf{e_R}$, $x^i = (r,\ \theta,\ \phi)$, and $X^i = (R,\ \Theta,\ \Phi)$, where R is the radius of the wall and $\dot{R} \equiv \frac{dR}{dt}$ is the wall velocity. We also have used $h^{i0} = 0$ in the derivation of above equations. Thus, the motion of a spherical bubble wall is described by Eq.(3.8).

Similarly, for a spherical dust shell ($\tau = 0$), the jump condition and the equation of motion are given by

$$\left[w\gamma^2 v\right]_{R_-}^{R^+} + \frac{1}{R^2\dot{R}}\frac{d}{dt}(\Gamma^3\sigma R^2\dot{R}) = \chi\frac{\partial T}{\partial r} + \xi\frac{\partial(\gamma v)}{\partial t} \tag{3.9}$$

and

$$\left[w\gamma^2 v^2 + p\right]_{R_-}^{R^+} + \frac{1}{R^2\dot{R}}\frac{d}{dt}(\Gamma^3\sigma R^2\dot{R}^2) = (\frac{1}{3}\eta + \xi)\frac{\partial(\gamma v)}{\partial r}. \tag{3.10}$$

4. Application to the QCD Phase Transition

In this section, we consider, specifically, a cosmological quark-hadron (QCD) phase transition (Fig.1), which likely occurs $\sim 10\mu s$ after the Big Bang, as an application of the previous sections.

4A - QCD formalism

We use subscripts q and h to denote the quark and hadron phases, respectively and we assume a simple "bag model".[10]. The bag equations of state for each phase can then be written as

Quark phase:

$$e_q = \frac{\pi^2}{30}g_q T_q^4 + B, \quad p_q = \frac{\pi^2}{90}g_q T_q^4 - B,$$

$$s_q = \frac{2\pi^2}{45}g_q T_q^3, \quad g_q = 51.25. \tag{4.1}$$

Hadron phase:

$$e_h = \frac{\pi^2}{30}g_h T_h^4, \quad p_q = \frac{\pi^2}{90}g_h T_h^4,$$

$$s_q = \frac{2\pi^2}{45}g_h T_h^3, \quad g_h = 17.25. \tag{4.2}$$

where B is a bag constant, s represents entropy, and $g_{q\,\mathrm{or}\,h}$ measures the effective number of helicity states in the corresponding phases. In heavy ion collisions, the central collision volume is much smaller than the mean free paths of electromagnetically and weakly interacting particles, so only strongly interacting particles contribute to the pressure. Thus, $g_q = 37$ ($\frac{7}{8} \times 2\,\text{quarks} \times 2\,\text{spin states} \times 3\,\text{colors} + \frac{7}{8} \times 2\,\text{antiquarks} \times 2\,\text{spin states} \times 3\,\text{colors} + 8\,\text{gluons} \times 2\,\text{helicities}$) and $g_h = 3$ (3 pions). However, for bubbles in the early universe much larger than the mean free path of a neutrino ($\lambda_\nu \sim 1\text{cm}$), we must add the electroweak degrees of freedom ($\frac{7}{8} \times 2\,\text{charged leptons} \times 2\,\text{spin states} + \frac{7}{8} \times 2\,\text{anti charged leptons} \times 2 + \frac{7}{8} \times 3\,\text{neutrinos} \times 2\,\text{helicities}) + 1\,\text{photon} \times 2\,\text{helicities}$), so $g_q = 51.25$ and $g_h = 17.25$.

There are two possibilities for this transition: First, if it is a second order phase transition, our jump formalism is obviously not appropriate (of course if there is no phase transition at all, the irrelevance of our arguments is also true.), but then the bag assumptions also fail to apply. However, if the QCD transition is a first order phase transition, then we have

$$e_q = e_h, \qquad p_q = p_h, \tag{4.5}$$

which implies that

$$\left(\frac{T_h}{T_q}\right)^4 = \frac{g_q}{g_h} - \frac{3}{a g_h}\frac{B}{T_q^4}. \tag{4.6a}$$

For the given values of g_q and g_h (Eqs. (4.1) and (4.2)), it turns into

$$\left(\frac{T_h}{T_q}\right)^4 = 2.97 - 0.52\frac{B}{T_q^4}. \tag{4.6b}$$

It is clear from this expression that, we can always find a point to satisfy the above equations, including the point $T_q = T_h = T_c$. Therefore, we conclude that a consistent first order QCD cosmological phase transition is possible if g_q is greater than g_h as is assumed in Eqs.(4.1) and (4.2).

If this transition is actually first order, we can make the following remarks:

i) An upper limit for the bag constant B:

Physically, the transition should occur when the temperature of the quark phase falls below the mass of a meson (e.g., m_π). Furthermore, it is always true mathematically that $\left(\frac{T_h}{T_q}\right)^4 \geq 0$. Combining these two requirements, we obtain a natural upper limit for the bag constant to fit these assumptions:

$$B^{1/4} \leq \left(\frac{a g_q}{3}\right)^{1/4} m_\pi \simeq 215\,\text{MeV}. \tag{4.7}$$

where we have used the values of g_q and g_h given in Eqs.(4.1) and (4.2). This result is consistent with the so called MIT bag model($B^{1/4} = 146$MeV),[11] but contrary to the parameter values used in the chiral bag model($B^{1/4} = 276$MeV). [12]

ii) Unnecessary thermal equilibrium:

Now we simply express the necessary conditions (Eq. (4.6a,b)) for a first order phase transition in Figure 2. We see that the locus of all of the points ($\frac{T_h}{T_q}$) described by Eq.(4.6) defines a curve, any point on which can be the transition point; the thermal equilibrium transition $T_q = T_h = T_c$ is only one of them. We are thus led to the conclusion that, in general, for a first order transition, a complete thermal equilibrium between the two phases is not necessary; there may exist a small temperature difference between the phases, such that the phase boundary moves from the new (hadron) phase toward the old (quark) phase. The magnitude of the difference depends on the bag constant. For $B^{1/4} \simeq 192\,\text{MeV}$(which corresponds to hadrons with a radius of 0.7 fm), $T_h/T_q \sim 1.03$, which is near a thermal equilibrium transition, but has a supercooling of $\sim 3\%$.

4B - QCD jump conditions

For a first order QCD phase transition, the hadron bubble is just a domain wall. If it is spherical, the jump conditions are given by

$$w_q\gamma^2 v_q - w_h\gamma^2 v_h = \chi\frac{\partial T}{\partial r} + \xi\frac{\partial(\gamma v)}{\partial t} \tag{4.8}$$

and

$$w_q\gamma^2 v_q^2 - w_h\gamma^2 v_h^2 + p_q - p_h + \frac{1}{R^2\dot{R}}\frac{d}{dt}(\Gamma\sigma R^2) = (\frac{1}{3}\eta + \xi)\frac{\partial(\gamma v)}{\partial r}. \tag{4.9}$$

At zero temperature ($T = 0$), Eq.(4.9) reads

$$4\pi R^2\Gamma\sigma = \frac{4}{3}\pi R^3 B, \tag{4.10}$$

which is Coleman's well-known solution.[13]

Moreover, if the viscosities (shear and bulk) are small and negligible on this energy scale during the phase transition, the above jump conditions have the simple form

$$w_q\gamma^2 v_q - w_h\gamma^2 v_h = \chi\frac{\partial T}{\partial r} \tag{4.11}$$

and

$$w_q\gamma^2 v_q^2 + p_q + \frac{1}{R^2\dot{R}}\frac{d}{dt}(\Gamma\sigma R^2) = w_h\gamma^2 v_h^2 + p_h. \tag{4.12}$$

4C - QCD energy transport

There are two principle ways for energy to be transported during this phase transition: (1) by hydrodynamic flow, i.e., $F_H = w_h\gamma^2 v_h$;[14] and (2) by neutrino radiation or conduction, i.e., $F_\nu = \frac{a}{4} g_\nu (T_q^4 - T_h^4) \simeq a g_\nu T_q^4 \frac{\Delta T}{T_q}$,[15] where g_ν is the neutrino degeneracy factor and $\Delta T = T_q - T_h$. We now consider the simple case (no viscosity and no heat conductivity) and explore the critical conditions for either mode to be dominant.

For the purpose of simplification, we introduce the surface term

$$S = \frac{1}{\dot{R} R^2} \frac{d}{dt} \left(\frac{R^2 \sigma}{\sqrt{1 - \dot{R}^2}} \right). \tag{4.13}$$

If $\sigma = \text{const}$, S will assume a simple form

$$S = \frac{\sigma}{(1 - \dot{R}^2)^{3/2}} \left(\frac{2}{R} - 2\frac{\dot{R}^2}{R} + \frac{d\dot{R}}{dt} \right). \tag{4.14}$$

At low velocity, the energy flux carried by hydrodynamic flow can be obtained from Eqs. (4.11) and (4.12)

$$F_H = \frac{4}{3} a \sqrt{g_q g_h}\, T_q^4 \left[\frac{1}{4} + \frac{S - B}{Q} \right]^{1/2} \left(1 - \frac{\Delta T}{T_q}\right)^2, \tag{4.15}$$

where $Q = w_q - w_h = 4B$ is the latent heat of the transition.

Thus, the ratio of the neutrino energy flux to the hydrodynamic energy flux is

$$\frac{F_v}{F_H} = \frac{3}{2} \sqrt{\frac{g_\nu^2}{g_q g_h}} \frac{\Delta T / T_q}{\left(\frac{S}{B}\right)^{1/2} \left(1 - \frac{\Delta T}{T_q}\right)^2} \sim 0.4 \sqrt{\frac{B}{S}} \frac{\frac{\Delta T}{T_q}}{1 - 2\frac{\Delta T}{T_q}}, \tag{4.16}$$

where we have used the approximation $\left(1 - \frac{\Delta T}{T_q}\right)^2 \sim 1 - 2\frac{\Delta T}{T_q}$ and Eqs.(4.1) and (4.2) for g_q and g_h. For a slowly moving boundary, $S \simeq 2\sigma/R$, the ratio becomes

$$\frac{F_v}{F_H} = 0.283 \sqrt{\frac{B}{\sigma}}\, R^{1/2} \frac{\frac{\Delta T}{T_q}}{1 - 2\frac{\Delta T}{T_q}}. \tag{4.17}$$

From this expression, we can clearly see that, if the hadron bubble is very small (i.e., R is small), the surface pressure will be very strong, independent of the magnitude of supercooling ($\frac{\Delta T}{T_q}$), and the ratio of the two energy fluxes will always remain very small, even approaching zero, i.e., $\frac{F_\nu}{F_H} \to 0$ (note: the energy carried by neutrinos is always limited by the amplitude of supercooling). On the other hand, if the bubble is large, the surface pressure will become small possibly even negligible; in particular, for large values of R, the ratio could become large– much greater than one.

We can therefore now conclude that, in the early stage of growth of the hadron bubble or the phase transition (R is small), energy transport is dominated by the hydrodynamic flow. As the bubbles get bigger and attain a certain critical size at which the surface pressure becomes negligible, the hydrodynamic flow will be limited by neutrino diffusion[14]. At this stage, the energy flux carried by neutrinos will take over and become dominant.

The critical condition for neutrino conduction to become compatible with the hydrodynamic flow is obtained by setting the ratio $\frac{F_\nu}{F_H} \geq 1$, i.e.,

$$R \geq 12.5\,\frac{\sigma}{B}\Big(\frac{\frac{\Delta T}{T_q}}{1-2\frac{\Delta T}{T_q}}\Big)^{-2} \sim 3.54\,\Big(\frac{\frac{\Delta T}{T_q}}{1-2\frac{\Delta T}{T_q}}\Big)^{-2}\ \mathrm{fm}. \tag{4.18}$$

where we have used $\sigma = 50\,\mathrm{MeV/fm^2}, B^{1/4} = 192\,\mathrm{MeV}$, and $\sigma/B = 0.283\,\mathrm{fm}$. Alternatively, in view of the physics, neutrino conduction becomes efficient when (and only when) the size of the bubble is larger than the neutrino mean free path $\lambda_\nu = G_F^{-2}T_c^{-5} \sim 1\,\mathrm{cm}$,[16] where G_F is the weak coupling constant. Thus, we can calculate the amount of the supercooling at that time as

$$\frac{\Delta T}{T_q} \leq \sqrt{\frac{3.54\times 10^{-13}}{\lambda_\nu}} \sim 6\times 10^{-6}. \tag{4.19}$$

The maximum initial supercooling can be estimated by setting R equal to the critical radius of the nucleating hadron bubble $R_{\mathrm{cr}} = 2\sigma/(p_h - p_q)$, i.e., the minimum radius required for a bubble to grow rather than shrink away,

$$\left[\frac{\Delta T}{T_q}\right]_0 \leq \frac{\beta\,\frac{F_\nu}{F_H}}{1+2\beta\,\frac{F_\nu}{F_H}} \ll \beta \sim 45\%, \tag{4.20}$$

where $\beta \equiv \left(\frac{3.54\times10^{-13}}{R_{\mathrm{cr}}}\right)^{1/2}$ and $R_{\mathrm{cr}} \simeq 17.3\,\mathrm{fm}$ for the parameters we use, and $F_\nu \ll F_H$. If the flux F_ν is initially compatible with F_H, the initial supercooling will be $\sim 24\%$. This result is consistent with the recent result obtained in Ref.[17].

For a first order QCD phase transition, the large supercooling can lead to important consequences on big bang nucleosynthesis. The major potential effects are possible large nucleation and creation of baryonic density and neutron to proton ratio inhomogeneities in cosmic matter[18]. It is interesting that these inhomogeneities can significantly modify the abundances of light-elements predicted by standard model[19] unless the average baryon density is near that required by homogeneous Big Bang Nucleosynthesis[20]. As noted by Appligate, Hogan and Scherrer[21], the result will be that the nucleosynthesis in the high-density region occurs with a low $\frac{n}{p}$ while the low-density region has a high $\frac{n}{p}$. Regions with $\frac{n}{p} > 1$ have qualitavely different nucleosynthesis than standard homogeneous nucleosynthesis (where $\frac{n}{p} \sim \frac{1}{7}$). Specially, If $\frac{n}{p} > 1$, the number of protons rather than neutrons becomes the constraining parameter on the reaction network flow toward ^4He. Meanwhile, an exciting possibility might arise: i.e., the baryonic density fluctuation might enable big bang nucleosynthesis to make elements heavier than ^7Li which are blocked in the conventional model. However, Thomas et al [22] have shown that such high A leakage is not significant for parameter values that fit the $A \leq 7$ abundances.

The application of this formalism to an ultrarelativistic heavy ion collision should be straightforward. In heavy ion collisions, where large chemical potential and dissipative processes exist and the surface effects are apparently important, the energy transport during the phase transition will be dominated by hydrodynamic flow rather than radiation. In the meanwhile, the shape and the velocity of the propagating bubble wall should be essentially affected as well. We intend to quantitatively explore this in future work.

5. Conclusions and Discussion

In this paper, we have derived covariant jump conditions across a bubble wall, including surface tension and possible dissipation, for a surface moving in any manner but without gravity. Special cases, such as those associated with domain walls, dust shells, and spherical boundary conditions, are also discussed. However one of the main applications is the cosmological QCD phase transition. The possible mechanism of energy transport during the phase transition is studied. A critical condition for neutrino conduction to dominate over hydrodynamic flow is obtained, with the following result: hydrodynamic flow dominates only when the bubble is small. As the bubbles grow and reach a large size, neutrino conduction becomes dominant. The critical size of the bubble depends on the amount of the initial supercooling. If the initial supercooling is very small, neutrino conduction can become dominant only when the size of the bubble is greater than the mean free path of the neutrinos ($\lambda_\nu \sim 1\text{cm}$). However, for large initial supercooling, neutrino conduction could become dominant as soon as the bubble attains its critical size, which can even be less than λ_ν. Moreover, the magnitude of the supercooling in the transition is estimated. Using this formalism argues for bag constants $B^{1/4} \leq 215\,\text{MeV}$ in order to be consistent through such a first order phase transition. Models with values outside this range, while still empirically interesting, are not self consistent with the first order transition that Bag models implicitly assume.

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Figure Captions

Fig.1 Phase diagram of the Quark-Hadron phase transition.

Fig.2 The temperatures of two phases in the first order phase transition.

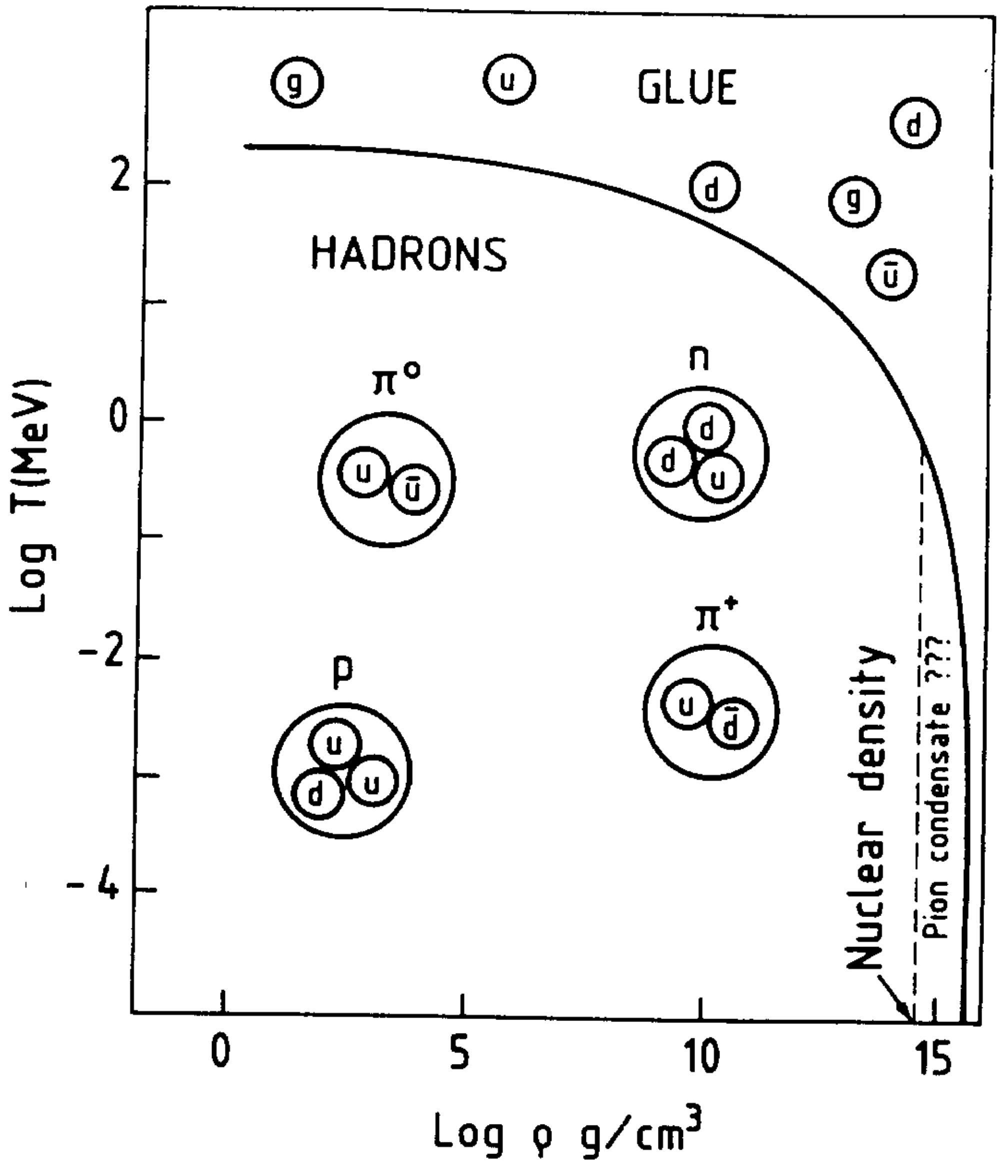

Fig.1 Phase diagram of the Quark-Hadron phase transition.

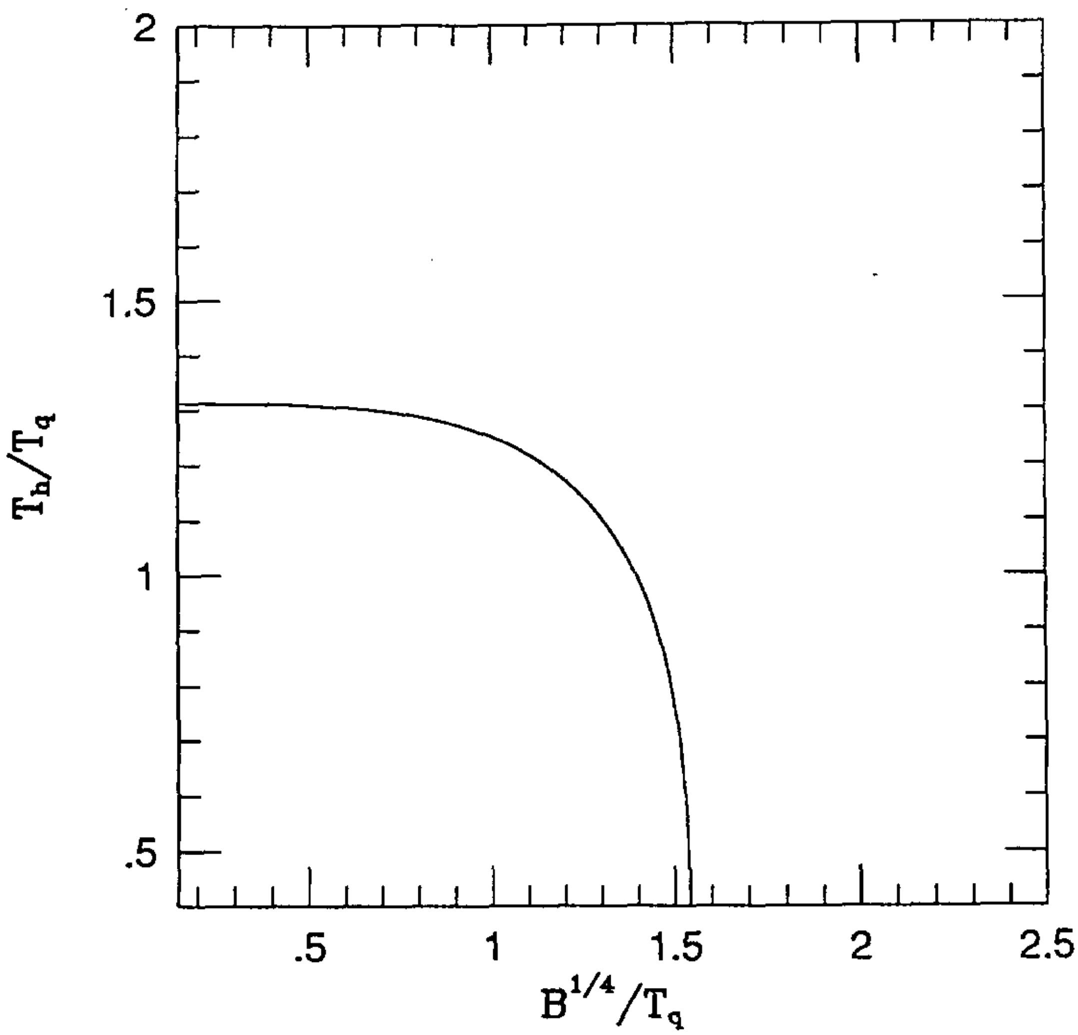

Fig.2 The temperatures of two phases during the transition.